

A magnifying glass with a wooden handle and a metal spring mechanism is positioned over a white surface. The lens of the magnifying glass is centered on the text, which is displayed in a large, bold, black serif font. The background is a plain, light-colored surface.

Lenses, Folds, and Traversals

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Lenses

Overview

What is a Lens?

“The Power is in the Dot”

Semantic Editor Combinators

Setters

Traversals

Folds

Lenses

Getters

Overloading Application

Extras

Uniplate

Zippers



Lenses

What is a Lens?

What is a Lens?

Lenses



What is a Lens?

Costate Comonad Coalgebra
is equivalent of Java's
member variable update
technology for Haskell



@PLT_Borat

Lenses

What is a Lens?

```
data Lens s a = Lens { set    :: s -> a -> s
                      , view  :: s -> a
                      }
```

```
view :: Lens s a -> s -> a
set   :: Lens s a -> s -> a -> s
```

Laws:

- 1.) `set l (view l s) s = s`
- 2.) `view l (set l s a) = a`
- 3.) `set l (set l s a) b = set l s b`

Lenses

What is a Lens?

```
data Lens s a = Lens { set    :: s -> a -> s
                      , view  :: s -> a
                      }
```

Lenses

What is a Lens?

```
data Lens s a = Lens { set    :: s -> a -> s
                      , view  :: s -> a
                      }
```

Fusion (and data-lens).

```
data Lens s a = Lens (s -> (a -> s, a))
```

Lenses

What is a Lens?

```
data Lens s a = Lens { set    :: s -> a -> s
                      , view  :: s -> a
                      }
```

Fusion (and `data-lens`).

```
data Lens s a = Lens (s -> (a -> s, a))
```

```
data Store s a = Store (s -> a) s
```

```
data Lens s a = Lens (s -> Store a s)
```


Lenses

What is a Lens?

```
newtype Lens s a = Lens (s -> Store a s)
```

```
data Store s a = Store (s -> a) s
```

```
instance Category Lens where
```

```
  id = Lens (Store id)
```

```
  Lens f . Lens g = Lens $ \r -> case g r of
```

```
    Store sr s -> case f s of
```

```
      Store ts t -> Store (sr . ts) t
```



Lenses

The Power is in the Dot

The Power is in the Dot

Lenses

The Power is in the Dot

(.) :: (a -> b) -> (c -> a) -> c -> b

Lenses

The Power is in the Dot

$(.) \quad :: (a \rightarrow b) \rightarrow (c \rightarrow a) \rightarrow b$

Lenses

The Power is in the Dot

$(.)$ $:: (a \rightarrow b) \rightarrow (c \rightarrow$ $a) \rightarrow c$ $\rightarrow b$
 $(.) . (.)$ $:: (a \rightarrow b) \rightarrow (c \rightarrow d \rightarrow$ $a) \rightarrow c \rightarrow d$ $\rightarrow b$

Lenses

The Power is in the Dot


$(.) :: (a \rightarrow b) \rightarrow (c \rightarrow a) \rightarrow c \rightarrow b$
 $(.).(.) :: (a \rightarrow b) \rightarrow (c \rightarrow d \rightarrow a) \rightarrow c \rightarrow d \rightarrow b$
 $(.).(.).(.) :: (a \rightarrow b) \rightarrow (c \rightarrow d \rightarrow e \rightarrow a) \rightarrow c \rightarrow d \rightarrow e \rightarrow b$

Lenses

The Power is in the Dot

```
(.)           :: (a -> b) -> (c -> a) -> c -> b
(.).(.)      :: (a -> b) -> (c -> d -> a) -> c -> d -> b
(.).(.).(.)  :: (a -> b) -> (c -> d -> e -> a) -> c -> d -> e -> b
```

```
fmap          :: Functor f          => (a -> b) -> f a -> f b
fmap.fmap     :: (Functor f, Functor g) => (a -> b) -> f (g a) -> f (g b)
fmap.fmap.fmap :: (Functor f, Functor g, Functor h) => (a -> b) -> f (g (h a)) -> f (g (h b))
```



Lenses

Semantic Editor Combinators

Semantic Editor Combinators

Lenses

Semantic Editor Combinators

```
(.)           :: (a -> b) -> (c -> a) -> c -> b
(.).(.)      :: (a -> b) -> (c -> d -> a) -> c -> d -> b
(.).(.).(.)  :: (a -> b) -> (c -> d -> e -> a) -> c -> d -> e -> b
```

```
fmap          :: Functor f          => (a -> b) -> f a -> f b
fmap.fmap     :: (Functor f, Functor g) => (a -> b) -> f (g a) -> f (g b)
fmap.fmap.fmap :: (Functor f, Functor g, Functor h) => (a -> b) -> f (g (h a)) -> f (g (h b))
```

These are sometimes known as **Semantic Editor Combinators**.

```
result = (.)
```

```
element = fmap
```

```
second = fmap
```

```
first f (a,b) = (f a, b)
```

Lenses

Semantic Editor Combinators

```
(.)      :: (a -> b) -> (c -> a) -> c -> b
(.).(.)  :: (a -> b) -> (c -> d -> a) -> c -> d -> b
(.).(.).(.) :: (a -> b) -> (c -> d -> e -> a) -> c -> d -> e -> b
```

```
fmap      :: Functor f      => (a -> b) -> f a      -> f b
fmap.fmap :: (Functor f, Functor g) => (a -> b) -> f (g a) -> f (g b)
fmap.fmap.fmap :: (Functor f, Functor g, Functor h) => (a -> b) -> f (g (h a)) -> f (g (h b))
```

These are sometimes known as **Semantic Editor Combinators**.

```
type SEC s t a b = (a -> b) -> s -> t
```

```
result :: SEC (e -> a) (e -> b) a b
```

```
result = (.)
```

```
element :: SEC [a] [b] a b
```

```
element = fmap
```

```
second :: SEC (c,a) (c,b) a b
```

```
second = fmap
```

```
first :: SEC (a,c) (b,c) a b
```

```
first f (a,b) = (f a, b)
```

Lenses

Semantic Editor Combinators

```
(.)      :: (a -> b) -> (c -> a) -> c -> b
(.).(.)  :: (a -> b) -> (c -> d -> a) -> c -> d -> b
(.).(.).(.) :: (a -> b) -> (c -> d -> e -> a) -> c -> d -> e -> b
```

```
fmap      :: Functor f      => (a -> b) -> f a      -> f b
fmap.fmap :: (Functor f, Functor g) => (a -> b) -> f (g a) -> f (g b)
fmap.fmap.fmap :: (Functor f, Functor g, Functor h) => (a -> b) -> f (g (h a)) -> f (g (h b))
```

These are sometimes known as **Semantic Editor Combinators**.

```
type SEC s t a b = (a -> b) -> s -> t
```

```
fmap :: Functor f => SEC (f a) (f b) a b
```

```
first :: SEC (a,c) (b,c) a b
first f (a,b) = (f a, b)
```



Setters

Lenses

Setters

```
(.)      :: (a -> b) -> (c -> a) -> c -> b
(.).(.)  :: (a -> b) -> (c -> d -> a) -> c -> d -> b
(.).(.).(.) :: (a -> b) -> (c -> d -> e -> a) -> c -> d -> e -> b
```

```
fmap      :: Functor f      => (a -> b) -> f a      -> f b
fmap.fmap :: (Functor f, Functor g) => (a -> b) -> f (g a) -> f (g b)
fmap.fmap.fmap :: (Functor f, Functor g, Functor h) => (a -> b) -> f (g (h a)) -> f (g (h b))
```

```
traverse      :: (Traversable f, Applicative m)
=> (a -> m b) -> f a      -> m (f b)
traverse.traverse :: (Traversable f, Traversable g, Applicative m)
=> (a -> m b) -> f (g a) -> m (f (g b))
traverse.traverse.traverse :: (Traversable f, Traversable g, Traversable h, Applicative m)
=> (a -> m b) -> f (g (h a)) -> m (f (g (h b)))
```

Lenses

Setters

```
class Functor f where  
  fmap :: (a -> b) -> f a -> f b
```

```
class (Functor f, Foldable f) => Traversable f where  
  traverse :: Applicative m => (a -> m b) -> f a -> m (f b)  
  ...
```

Lenses

Setters

```
fmap      :: Functor f      => (a -> b) -> f a -> f b
```

```
traverse :: (Traversable f, Applicative m) => (a -> m b) -> f a -> m (f b)
```

Lenses

Setters

```
fmap      :: Functor f      => (a -> b) -> f a -> f b  
fmapDefault :: Traversable f => (a -> b) -> f a -> f b
```

```
traverse :: (Traversable f, Applicative m) => (a -> m b) -> f a -> m (f b)
```


Lenses

Setters

```
fmap      :: Functor f      => (a -> b) -> f a -> f b
fmapDefault :: Traversable f => (a -> b) -> f a -> f b
fmapDefault f = runIdentity . traverse (Identity . f)
```

```
traverse :: (Traversable f, Applicative m) => (a -> m b) -> f a -> m (f b)
```

```
newtype Identity a = Identity { runIdentity :: a }
```

```
instance Functor Identity where
  fmap f (Identity a) = Identity (f a)
```

```
instance Applicative Identity where
  pure = Identity
  Identity f <*> Identity x = Identity (f x)
```

Lenses

Setters

```
fmap           :: Functor f           => (a -> b) -> f a           -> f b
fmap.fmap      :: (Functor f, Functor g) => (a -> b) -> f (g a)       -> f (g b)
fmap.fmap.fmap :: (Functor f, Functor g, Functor h) => (a -> b) -> f (g (h a)) -> f (g (h b))
```

```
fmap           :: Functor f           => (a -> b) -> f a -> f b
fmapDefault    :: Traversable f       => (a -> b) -> f a -> f b
fmapDefault f = runIdentity . traverse (Identity . f)
```

```
traverse :: (Traversable f, Applicative m) => (a -> m b) -> f a -> m (f b)
```

```
newtype Identity a = Identity { runIdentity :: a }
```

```
instance Functor Identity where
  fmap f (Identity a) = Identity (f a)
```

```
instance Applicative Identity where
  pure = Identity
  Identity f <*> Identity x = Identity (f x)
```

Lenses

Setters

```
fmap          :: Functor f          => (a -> b) -> f a          -> f b
fmap.fmap     :: (Functor f, Functor g) => (a -> b) -> f (g a)      -> f (g b)
fmap.fmap.fmap :: (Functor f, Functor g, Functor h) => (a -> b) -> f (g (h a)) -> f (g (h b))
```

```
fmapDefault :: Traversable f => (a -> b) -> f a -> f b
fmapDefault = runIdentity . traverse (Identity . f)
```

```
over l f = runIdentity . l (Identity . f)
```

Lenses

Setters

```
fmap           :: Functor f           => (a -> b) -> f a           -> f b
fmap.fmap      :: (Functor f, Functor g) => (a -> b) -> f (g a)       -> f (g b)
fmap.fmap.fmap :: (Functor f, Functor g, Functor h) => (a -> b) -> f (g (h a)) -> f (g (h b))
```

```
fmapDefault :: Traversable f => (a -> b) -> f a -> f b
fmapDefault = runIdentity . traverse (Identity . f)
```

```
over 1 f = runIdentity . 1 (Identity . f)
```

```
over traverse f = runIdentity . traverse (Identity . f)
                 = fmapDefault f
                 = fmap f
```

Lenses

Setters

```
fmap          :: Functor f          => (a -> b) -> f a          -> f b
fmap.fmap     :: (Functor f, Functor g) => (a -> b) -> f (g a)      -> f (g b)
fmap.fmap.fmap :: (Functor f, Functor g, Functor h) => (a -> b) -> f (g (h a)) -> f (g (h b))
```

```
fmapDefault :: Traversable f => (a -> b) -> f a -> f b
fmapDefault = runIdentity . traverse (Identity . f)
```

```
over :: (a -> Identity b) -> s -> Identity t) -> (a -> b) -> s -> t
over l f = runIdentity . l (Identity . f)
```

```
over traverse f = runIdentity . traverse (Identity . f)
                = fmapDefault f
                = fmap f
```

Lenses

Setters

```
fmap          :: Functor f          => (a -> b) -> f a          -> f b
fmap.fmap     :: (Functor f, Functor g) => (a -> b) -> f (g a)      -> f (g b)
fmap.fmap.fmap :: (Functor f, Functor g, Functor h) => (a -> b) -> f (g (h a)) -> f (g (h b))
```

```
fmapDefault :: Traversable f => (a -> b) -> f a -> f b
fmapDefault = runIdentity . traverse (Identity . f)
```

```
over :: Setter s t a b -> (a -> b) -> s -> t
over l f = runIdentity . l (Identity . f)
```

```
over traverse f = runIdentity . traverse (Identity . f)
                = fmapDefault f
                = fmap f
```

```
type Setter s t a b = (a -> Identity b) -> s -> Identity t
```

Lenses

Setters

```
mapped          :: Functor f => Setter (f a) (f b) a b
mapped.mapped   :: (Functor f, Functor g) => Setter (f (g a)) (f (g b)) a b
mapped.mapped.mapped :: (Functor f, Functor g, Functor h) => Setter (f (g (h a))) (f (g (h b))) a b
```

```
fmapDefault :: Traversable f => (a -> b) -> f a -> f b
fmapDefault = runIdentity . traverse (Identity . f)
```

```
over :: Setter s t a b -> (a -> b) -> s -> t
over l f = runIdentity . l (Identity . f)
```

```
over traverse f = runIdentity . traverse (Identity . f)
                = fmapDefault f
                = fmap f
```

```
type Setter s t a b = (a -> Identity b) -> s -> Identity t
```

```
mapped :: Functor f => Setter (f a) (f b) a b
mapped f = Identity . fmap (runIdentity . f)
```

```
over mapped f = runIdentity . mapped (Identity . f)
              = runIdentity . Identity . fmap (runIdentity . Identity . f)
              = fmap f
```

Lenses

Setters

```
mapped          :: Functor f => Setter (f a) (f b) a b
mapped.mapped   :: (Functor f, Functor g) => Setter (f (g a)) (f (g b)) a b
mapped.mapped.mapped :: (Functor f, Functor g, Functor h) => Setter (f (g (h a))) (f (g (h b))) a b
```

```
over :: Setter s t a b -> (a -> b) -> s -> t
```

```
over mapped :: Functor f => (a -> b) -> f a -> f b
over (mapped.mapped) :: (Functor f, Functor g) => (a -> b) -> f (g a) -> f (g b)
...
```

```
chars :: (Char -> Identity Char) -> Text -> Identity Text
chars f = fmap pack . mapped f . unpack
```

```
over chars :: (Char -> Char) -> Text -> Text
over (mapped.chars) :: Functor f => (Char -> Char) -> f Text -> f Text
over (traverse.chars) :: Traversable f => (Char -> Char) -> f Text -> f Text
```


Lenses

Setters

```
mapped          :: Functor f => Setter (f a) (f b) a b
mapped.mapped   :: (Functor f, Functor g) => Setter (f (g a)) (f (g b)) a b
mapped.mapped.mapped :: (Functor f, Functor g, Functor h) => Setter (f (g (h a))) (f (g (h b))) a b
```

```
over :: Setter s t a b -> (a -> b) -> s -> t
```

```
over mapped :: Functor f => (a -> b) -> f a -> f b
over (mapped.mapped) :: (Functor f, Functor g) => (a -> b) -> f (g a) -> f (g b)
...
```

```
chars :: (Char -> Identity Char) -> Text -> Identity Text
chars f = fmap pack . mapped f . unpack
```

```
over chars :: (Char -> Char) -> Text -> Text
over (mapped.chars) :: Functor f => (Char -> Char) -> f Text -> f Text
over (traverse.chars) :: Traversable f => (Char -> Char) -> f Text -> f Text
```

Lenses

Setters

```
mapped          :: Functor f => Setter (f a) (f b) a b
mapped.mapped   :: (Functor f, Functor g) => Setter (f (g a)) (f (g b)) a b
mapped.mapped.mapped :: (Functor f, Functor g, Functor h) => Setter (f (g (h a))) (f (g (h b))) a b
```

```
over :: Setter s t a b -> (a -> b) -> s -> t
```

Functor Laws:

- 1.) `fmap id = id`
- 2.) `fmap f . fmap g = fmap (f . g)`

Setter Laws for a legal Setter *l*.

- 1.) `over l id = id`
- 2.) `over l f . over l g = over l (f . g)`

```
both :: Setter (a,a) (b,b) a b
both f (a,b) = (,) <$> f a <*> f b
```

```
first :: Setter (a,c) (b,c) a b
first f (a,b) = (,b) <$> f a
```

Lenses

Setters (are like Functors)

```
type Simple f s a = f s s a a
```

```
sets :: ((a -> b) -> s -> t) -> Setter s t a b
```

```
mapped :: Functor f => Setter (f a) (f b) a b
```

```
over, mapOf, (%~) :: Setter s t a b -> (a -> b) -> s -> t
```

```
set, (.~) :: Setter s t a b -> b -> s -> t
```

```
(+~), (-~), (*~) :: Num c => Setter s t c c -> c -> s -> t
```

```
(//~) :: Fractional c => Setter s t c c -> c -> s -> t
```

```
(||~), (&&~) :: Setter s t Bool Bool -> Bool -> s -> t
```

```
assign :: MonadState s m => Setter s s a b -> b -> m ()
```

```
(.=) :: MonadState s m => Setter s s a b -> b -> m ()
```

```
(%=) :: MonadState s m => Setter s s a b -> (a -> b) -> m ()
```

```
(+=), (-=), (*=) :: (MonadState s m, Num a) => Simple Setter s a -> a -> m ()
```

```
(//=) :: (MonadState s m, Fractional a) => Simple Setter s a -> a -> m ()
```

```
(||=), (&&~) :: MonadState s m => Simple Setter s Bool -> Bool -> m ()
```



Traversals

Lenses

Traversals

```
type Traversal s t a b = forall f. Applicative f => (a -> f b) -> s -> f t
type Setter s t a b   = (a -> Identity b) -> s -> Identity t
```

```
traverse          :: Traversable f => Traversal (f a) (f b) a b
traverse.traverse :: (Traversable f, Traversable g) => Traversal (f (g a)) (f (g b)) a b
```

```
over traverse f = runIdentity . traverse (Identity . f)
                = fmapDefault f
                = fmap f
```

Lenses

Traversals

```
type Traversal s t a b = forall f. Applicative f => (a -> f b) -> s -> f t
type Setter s t a b    = (a -> Identity b) -> s -> Identity t
```

```
mapM :: (Traversable f, Monad m) => (a -> m b) -> f a -> m (f b)
mapM f = unwrapMonad . traverse (WrapMonad . f)
```

```
mapMOf :: Monad m => Traversal s t a b -> (a -> m b) -> s -> m t
mapMOf l f = unwrapMonad . l (WrapMonad . f)
```

```
traverse :: Traversable f => Traversal (f a) (f b) a b
traverse.traverse :: (Traversable f, Traversable g) => Traversal (f (g a)) (f (g b)) a b
```

```
over traverse f = runIdentity . traverse (Identity . f)
                = fmapDefault f
                = fmap f
```

```
mapMOf traverse f = unwrapMonad . traverse (WrapMonad . f)
                  = mapM f
```

Lenses

Traversals

```
type Traversal s t a b = forall f. Applicative f => (a -> f b) -> s -> f t
type Setter s t a b   = (a -> Identity b) -> s -> Identity t
```

Traversable Laws:

- 1.) `traverse pure = pure`
- 2.) `Compose . fmap (traverse f) . traverse g ≡ traverse (Compose . fmap f . g)`

Laws for a valid Traversal `l`:

- 1.) `l pure = pure`
- 2.) `Compose . fmap (l f) . l g = l (Compose . fmap f . g)`

Lenses

Traversals

```
type Traversal s t a b = forall f. Applicative f => (a -> f b) -> s -> f t
type Setter s t a b    =                      (a -> Identity b) -> s -> Identity t
```

```
traverse :: Traversable f => Traversal (f a) (f b) a b
```

```
both :: Traversal (a,a) (b,b) a b
both f (a,b) = (,) <$> f a <*> f b
```

```
traverseLeft :: Traversal (Either a c) (Either b c) a b
traverseLeft f (Left a) = Left <$> f a
traverseLeft f (Right c) = pure (Right c)
```

```
traverseRight :: Traversal (Either c a) (Either c b) a b
traverseRight f (Left c) = pure (Left c)
traverseRight f (Right a) = Right <$> f a
```

```
traverseLeft.both :: Traversal (Either (a,a) c) (Either (b,b) c) a b
```

```
>>> over (traverseLeft.both) (+1) $ Left (2,3)
Left (3,4)
```

```
(.~) = over
traverseLeft.both .~ (+1) $ Left (2,3)
traverseLeft.both +~ 1 $ Left (2,3)
```


Lenses

Traversals (are like Traversables)

```
type LensLike f s t a b = (a -> f b) -> s -> f t
```

```
type Traversal s t a b = forall f. Applicative f => (a -> f b) -> s -> f t
```

```
traverse :: Traversable t => Traversal (t a) (t b) a b
```

```
ignored :: Traversal s s a b
```

```
traverseLeft :: Traversal (Either a c) (Either b c) a b
```

```
traverseRight :: Traversal (Either c a) (Either c b) a b
```

```
both :: Traversal (a, a) (b, b) a b
```

```
traverseOf :: LensLike f s t a b -> (a -> f b) -> s -> f t
```

```
forOf :: LensLike f s t a b -> s -> (a -> f b) -> f t
```

```
mapMOf :: LensLike (WrappedMonad m) s t a b -> (a -> m b) -> s -> m t
```

```
forMOf :: LensLike (WrappedMonad m) a b c d -> s -> (a -> m b) -> m t
```



Lenses

Folds

Folds

Lenses

Folds

```
class Functor f where  
  fmap :: (a -> b) -> f a -> f b
```

```
class Foldable f where  
  foldMap :: Monoid m => (a -> m) -> f a -> m  
  ...
```

```
class (Functor f, Foldable f) => Traversable f where  
  traverse :: Applicative m => (a -> m b) -> f a -> m (f b)  
  ...
```

Lenses

Folds

```
(.)      :: (a -> b) -> (c -> a)      -> c      -> b
(.).(.)  :: (a -> b) -> (c -> d -> a)  -> c -> d  -> b
(.).(.).(.) :: (a -> b) -> (c -> d -> e -> a) -> c -> d -> e -> b
```

```
fmap      :: Functor f      => (a -> b) -> f a      -> f b
fmap.fmap :: (Functor f, Functor g) => (a -> b) -> f (g a) -> f (g b)
fmap.fmap.fmap :: (Functor f, Functor g, Functor h) => (a -> b) -> f (g (h a)) -> f (g (h b))
```

```
foldMap      :: (Foldable f, Monoid m)
=> (a -> m) -> f a      -> m
foldMap.foldMap :: (Foldable f, Foldable g, Monoid m)
=> (a -> m) -> f (g a) -> m
foldMap.foldMap.foldMap :: (Foldable f, Foldable g, Foldable h, Monoid m)
=> (a -> m) -> f (g (h a)) -> m
```

Lenses

Folds

```
(.)      :: (a -> b) -> (c -> a)      -> c      -> b
(.).(.)  :: (a -> b) -> (c -> d -> a)  -> c -> d  -> b
(.).(.).(.) :: (a -> b) -> (c -> d -> e -> a) -> c -> d -> e -> b
```

```
fmap      :: Functor f      => (a -> b) -> f a      -> f b
fmap.fmap :: (Functor f, Functor g) => (a -> b) -> f (g a) -> f (g b)
fmap.fmap.fmap :: (Functor f, Functor g, Functor h) => (a -> b) -> f (g (h a)) -> f (g (h b))
```

```
foldMap      :: (Foldable f, Monoid m)
=> (a -> m) -> f a      -> m
foldMap.foldMap :: (Foldable f, Foldable g, Monoid m)
=> (a -> m) -> f (g a)  -> m
foldMap.foldMap.foldMap :: (Foldable f, Foldable g, Foldable h, Monoid m)
=> (a -> m) -> f (g (h a)) -> m
```

```
traverse      :: (Traversable f, Applicative m)
=> (a -> m b) -> f a      -> m (f b)
traverse.traverse :: (Traversable f, Traversable g, Applicative m)
=> (a -> m b) -> f (g a)  -> m (f (g b))
traverse.traverse.traverse :: (Traversable f, Traversable g, Traversable h, Applicative m)
=> (a -> m b) -> f (g (h a)) -> m (f (g (h b)))
```

Lenses

Folds

fmap :: **Functor** f => (a -> b) -> f a -> f b

foldMap :: (**Foldable** f, **Monoid** m) => (a -> m) -> f a -> m

traverse :: (**Traversable** f, **Applicative** m) => (a -> m b) -> f a -> m (f b)

Lenses

Folds

```
fmap      :: Functor f      => (a -> b) -> f a -> f b  
fmapDefault :: Traversable f => (a -> b) -> f a -> f b
```

```
foldMap      :: (Foldable f, Monoid m)    => (a -> m) -> f a -> m  
foldMapDefault :: (Traversable f, Monoid m) => (a -> m) -> f a -> m
```

```
traverse :: (Traversable f, Applicative m) => (a -> m b) -> f a -> m (f b)
```

Lenses

Folds

```
fmap      :: Functor f      => (a -> b) -> f a -> f b
fmapDefault :: Traversable f => (a -> b) -> f a -> f b
fmapDefault f = runIdentity . traverse (Identity . f)
```

```
foldMap      :: (Foldable f, Monoid m)      => (a -> m) -> f a -> m
foldMapDefault :: (Traversable f, Monoid m) => (a -> m) -> f a -> m
foldMapDefault f = getConst . traverse (Const . f)
```

```
traverse :: (Traversable f, Applicative m) => (a -> m b) -> f a -> m (f b)
```

```
newtype Const m a = Const { getConst :: m }
```

```
instance Functor (Const m) where
  fmap _ (Const m) = Const m
```

```
instance Monoid m => Applicative (Const m) where
  pure _ = Const mempty
  Const m <*> Const n = Const (m <> n)
```


Lenses

Folds

```
foldMap      :: (Foldable f, Monoid m)    => (a -> m) -> f a -> m
foldMapDefault :: (Traversable f, Monoid m) => (a -> m) -> f a -> m
foldMapDefault f = getConst . traverse (Const . f)
```

```
foldMapOf l f = getConst . l (Const . f)
```

```
folded :: Foldable f => Fold (f a) a
folded.folded :: (Foldable f, Foldable g) => Fold (f (g a)) a
folded.folded.folded :: (Foldable f, Foldable g, Foldable h) => Foldable (f (g (h a))) a
```

```
folded f = Const . foldMap (getConst . f)
```

```
foldMapOf folded f = getConst . Const . foldMap (getConst . Const . f)
                    = foldMap f
```

```
view l = getConst . l Const
```

```
view folded = getConst . Const . foldMap (getConst . Const)
              = foldMap id
              = fold
```

```
type Fold s a = forall m. (a -> Const m a) -> s -> Const m s
```

Lenses

Folds

```
foldMapOf l f = getConst . l (Const . f)
anyOf l f = getAny . foldMapOf l (Any . f)
sumOf l = getSum . foldMap Sum
```

```
>>> sumOf both (10,20)
30
```

```
>>> sumOf (traverse.both) [(10,20),(30,40)]
100
```



Lenses

More Than a Traversal

More Than a Traversal

Lenses

More Than a Traversal

```
type Traversal s t a b = forall f. Applicative f => (a -> f b)      -> s -> f t
```

```
type Setter s t a b      = (a -> Identity b) -> s -> Identity t  
type Fold s a            = forall m. Monoid m  => (a -> Const m a) -> s -> Const m s
```

```
view l = getConst . l Const
```

```
foldMapOf l f = getConst . l (Const . f)
```

```
toListOf l = getConst . l (\c -> Const [c])
```

```
_1 f (a,b) = (,b) <$> f a
```

```
>>> view _1 (10,20)  
10
```

```
_1 :: Traversal (a,c) (b,c) a b
```

```
>>> view _1 (10,20)  
No instance of Monoid for a  
No instance of Num for a
```

Lenses

More Than a Traversal

```
type Traversal s t a b = forall f. Applicative f => (a -> f b)      -> s -> f t
type Lens s t a b      = forall f. Functor f      => (a -> f b)      -> s -> f t
type Setter s t a b    =                          (a -> Identity b) -> s -> Identity t
type Fold s a          = forall m. Monoid m        => (a -> Const m a) -> s -> Const m s
type Getting r s t a b =                          (a -> Const r b) -> s -> Const r t
```

```
view :: Getting a s t a b -> s -> a
view l = getConst . l Const
```

```
foldMapOf :: Getting m s t a b -> (a -> m) -> s -> m
foldMapOf l f = getConst . l (Const . f)
```

```
toListOf :: Getting [a] s t a b -> s -> [a]
toListOf l = getConst . l (\a -> Const [a])
```

```
_1 :: Lens (a,c) (b,c) a b
_1 f (a,b) = (,b) <$> f a
```

```
>>> view _1 (10,20)
10
```

Lenses

More Than a Traversal

```
type Traversal s t a b = forall f. Applicative f => (a -> f b)      -> s -> f t
type Lens s t a b      = forall f. Functor f      => (a -> f b)      -> s -> f t
type Setter s t a b    =                          (a -> Identity b) -> s -> Identity t
type Fold s a          = forall m. Monoid m        => (a -> Const m a) -> s -> Const m s
type Getting r s t a b =                          (a -> Const r b) -> s -> Const r t
```

```
view :: Getting a s t a b -> s -> a
view l = getConst . l Const
```

```
set :: Setter s t a b -> b -> s -> t
set l d = runIdentity . l (Identity . const d)
```

```
lens :: (s -> a) -> (b -> s -> t) -> Lens s t a b
lens sa bst afb s = (`bst` s) <$> afb (sa s)
```

A Lens updates a *single* target. A Traversal can update (and read from) *many*.

A lens is just a valid traversal that targets a single element by only using the Functor out of whatever instance it is supplied.

Lenses

More Than a Traversal

```
type Traversal s t a b = forall f. Applicative f => (a -> f b)      -> s -> f t
type Lens s t a b      = forall f. Functor f      => (a -> f b)      -> s -> f t
type Setter s t a b    =                          (a -> Identity b) -> s -> Identity t
type Fold s a          = forall m. Monoid m        => (a -> Const m a) -> s -> Const m s
type Getting r s t a b =                          (a -> Const r b) -> s -> Const r t
```

```
_1 :: Lens (a,c) (b,c) a b
_1 f (a,b) = (,b) <$> f a
```

```
_2 :: Lens (c,a) (c,b) a b
_2 f (a,b) = (a,) <$> f b
```

```
at :: Ord k => k -> Simple Lens (Map k a) a
at
```

```
traverse._1 :: Applicative m => (a -> m b) -> [(a,c)] -> m [(a,c)]
```

```
>>> over (traverse._1) length $ [(“hello”,“San”),(“Francisco”,“:”)]
[(5,“San”),(9,“:”)]
```

```
view _1 (10,20)
10
```

Lenses

Folds (are like Foldables)

```
type Getting r s t a b = (a -> Const r b) -> s -> Const r t
```

```
(^?) :: s -> Getting (First a) s t a b -> Maybe a
```

```
(^..) :: s -> Getting [a] s t a b -> [a]
```

```
folding :: (Foldable f, Applicative g, Gettable g) => (s -> f a) -> LensLike g s t a b
```

```
folded :: Foldable f => Fold (f a) a
```

```
unfolded :: (s -> Maybe (a, s)) -> Fold s a
```

```
iterated :: (a -> a) -> Fold a a
```

```
backwards :: LensLike (Backwards f) s t a b -> LensLike f s t a b
```

```
repeated :: Fold a a
```

```
replicated :: Int -> Fold a a
```

```
takingWhile :: (Gettable f, Applicative f)
```

```
=> (a -> Bool) -> Getting (Endo (f s)) s s a a -> LensLike f s s a a
```

```
foldMapOf :: Getting r s t a b -> (a -> r) -> s -> r
```

```
foldOf :: Getting a s t a b -> s -> a
```

```
foldrOf :: Getting (Endo r) s t a b -> (a -> r -> r) -> r -> s -> r
```

```
toListOf :: Getting [a] s t a b -> s -> [a]
```

```
anyOf :: Getting Any s t a b -> (a -> Bool) -> s -> Bool
```

```
traverseOf_ :: Functor f => Getting (Traversed f) s t a b -> (a -> f r) -> s -> f ()
```




Getters

Lenses

Getters

```
type Traversal s t a b = forall f. Applicative f => (a -> f b)      -> s -> f t
type Lens s t a b      = forall f. Functor f      => (a -> f b)      -> s -> f t
type Setter s t a b    =                          (a -> Identity b) -> s -> Identity t
type Fold s a          = forall m. Monoid m        => (a -> Const m a) -> s -> Const m s
type Getting r s t a b =                          (a -> Const r b) -> s -> Const r t
type Getter s a        = forall r.                (a -> Const r a) -> s -> Const r s
```

```
to :: (s -> a) -> Getter s a
```

```
-- to :: (s -> a) -> (a -> Const r a) -> s -> Const r s
```

Lenses

Getters

```
type Traversal s t a b = forall f. Applicative f => (a -> f b)      -> s -> f t
type Lens s t a b      = forall f. Functor f      => (a -> f b)      -> s -> f t
type Setter s t a b    =                          (a -> Identity b) -> s -> Identity t
type Fold s a          = forall m. Monoid m        => (a -> Const m a) -> s -> Const m s
type Getting r s t a b =                          (a -> Const r b) -> s -> Const r t
type Getter s a        = forall r.                (a -> Const r a) -> s -> Const r s
```

```
to :: (s -> a) -> Getter s a
-- to :: (s -> a) -> (a -> Const r a) -> s -> Const r s
```

```
to_ :: (s -> a) -> (a -> r) -> s -> r
to_ f g = g . f
```

Lenses

Getters

```
type Traversal s t a b = forall f. Applicative f => (a -> f b)      -> s -> f t
type Lens s t a b      = forall f. Functor f      => (a -> f b)      -> s -> f t
type Setter s t a b    =                          (a -> Identity b) -> s -> Identity t
type Fold s a          = forall m. Monoid m        => (a -> Const m a) -> s -> Const m s
type Getting r s t a b =                          (a -> Const r b) -> s -> Const r t
type Getter s a        = forall r.                (a -> Const r a) -> s -> Const r s
```

```
to :: (s -> a) -> Getter s a
to f g = Const . getConst . g . f
```

```
view :: Getting a s t a b -> s -> a
view l = getConst . l Const
```

```
view (to f) = getConst . to f Const
            = getConst . Const . getConst . Const . f
            = f
```

Lenses

Getters

```
type Traversal s t a b = forall f. Applicative f => (a -> f b)      -> s -> f t
type Lens s t a b      = forall f. Functor f      => (a -> f b)      -> s -> f t
type Setter s t a b    =                          (a -> Identity b) -> s -> Identity t
type Fold s a          = forall m. Monoid m        => (a -> Const m a) -> s -> Const m s
type Getting r s t a b =                          (a -> Const r b) -> s -> Const r t
type Getter s a        = forall r.                (a -> Const r a) -> s -> Const r s
```

```
to :: (s -> a) -> Getter s a
to f g = Const . getConst . g . f
```

```
view :: Getting a s t a b -> s -> a
view l = getConst . l Const
```

```
view (to f) = getConst . to f Const
            = getConst . Const . getConst . Const . f
            = f
```

Lenses

Getters (are like functions)

```
class Functor f => Gettable f where  
  coerce :: f a -> f b
```

```
type Getter a c = forall f. Gettable f => (c -> f c) -> a -> f a  
newtype Accessor r a = Accessor { runAccessor :: r }
```

```
type Getting r a b c d = (c -> Accessor r d) -> a -> Accessor r b
```

```
to :: (s -> a) -> Getter s a  
to f g = coerce . g . f
```

```
(^.) :: s -> Getting a s t a b -> a  
view, (^!) :: Getting a s t a b -> s -> a
```

```
use :: MonadState s m => Getting a s t a b -> m a
```

Lenses

Lenses (Both Getter and Traversal)

```
type Lens s t a b = forall f. Functor f => (a -> f b) -> s -> f t
```

```
lens      :: (s -> a) -> (s -> b -> t) -> Lens s t a b
```

```
resultAt :: Eq e => e -> Simple Lens (e -> a) a
```

```
chosen   :: Lens (Either a a) (Either b b) a b
```

```
(<+~), (<-~), (<*~) :: Num a => LensLike ((,) a) s t a a -> a -> s -> (a, t)
```

```
(<//~)           :: Fractional a => LensLike ((,) a) s t a a -> a -> s -> (a, t)
```

```
(<||~), (<&&~)    :: LensLike ((,) Bool) s t Bool Bool -> Bool -> s -> (Bool, t)
```

```
(%%~)           :: LensLike f s t a b -> (a -> f b) -> s -> f t
```

```
(<+=), (<-=), (<*=) :: (MonadState s m, Num a) => SimpleLensLike ((,) a) s a -> a -> m a
```

```
(<//=)          :: (MonadState s m, Fractional a) => SimpleLensLike ((,) a) s a -> a -> m a
```

```
(<||=), (<&&=)    :: MonadState s m => SimpleLensLike ((,) Bool) s Bool -> Bool -> m Bool
```

```
(%%=)          :: MonadState s m => LensLike ((,) r) s s a b -> (a -> (r, b)) -> m r
```



Lenses

Fun Overloading

Isomorphisms

Lenses

Fun Overloading

```
class Category k => Isomorphic k where  
  isomorphic :: (a -> b) -> (b -> a) -> k a b
```

```
instance Isomorphic (->) where  
  isomorphic = const
```

```
data Isomorphism a b = Isomorphism (a -> b) (b -> a)
```

```
instance Category Isomorphism where  
  id = Isomorphism id id  
  Isomorphism bc cb . Isomorphism ab ba = Isomorphism (bc . ab) (ba . cb)
```

```
instance Isomorphic Isomorphism where  
  isomorphic = Isomorphism
```

```
type a <-> b = forall k. Isomorphic k => k a b
```

```
from :: Isomorphism a b -> b <-> a  
from (Isomorphism a b) = isomorphic b a
```

Lenses

Fun Overloading

```
class Category k => Isomorphic k where  
  isomorphic :: (a -> b) -> (b -> a) -> k a b
```

```
inc :: Num a => a -> a <-> a  
inc = isomorphic (+1) (subtract 1)
```

```
>>> inc 4  
5
```

```
>>> from inc 5  
4
```

Lenses

Fun Overloading

```
type Traversal s t a b = forall f. Applicative f      => (a -> f b) -> s -> f t
type Lens s t a b     = forall f. Functor f         => (a -> f b) -> s -> f t
type Iso s t a b      = forall k f. (Isomorphic k, Functor f) => k (a -> f b) (s -> f t)
```

```
iso :: (a -> b) -> (b -> a) -> Simple Iso a b
iso ab ba = isos ab ba ab ba
```

```
isos :: (a -> c) -> (c -> a) -> (b -> d) -> (d -> b) -> Iso a b c d
isos ac ca bd db = isomorphic (\cfd a -> db <$> cfd (ac a))
                             (\afb c -> bd <$> afb (ca c))
```

```
packed :: Simple Iso String Text
packed = iso pack unpack
```

```
text :: Simple Traversal Text Char
text = from packed . traverse
```

```
>>> anyOf (both.text) (=='c') ("chello"^packed,"world"^packed)
True
```



Lenses

Uniplate

Uniplate

Lenses

Uniplate

```
class Plated a where
  plate :: Simple Traversal a a
  plate = ignored
```

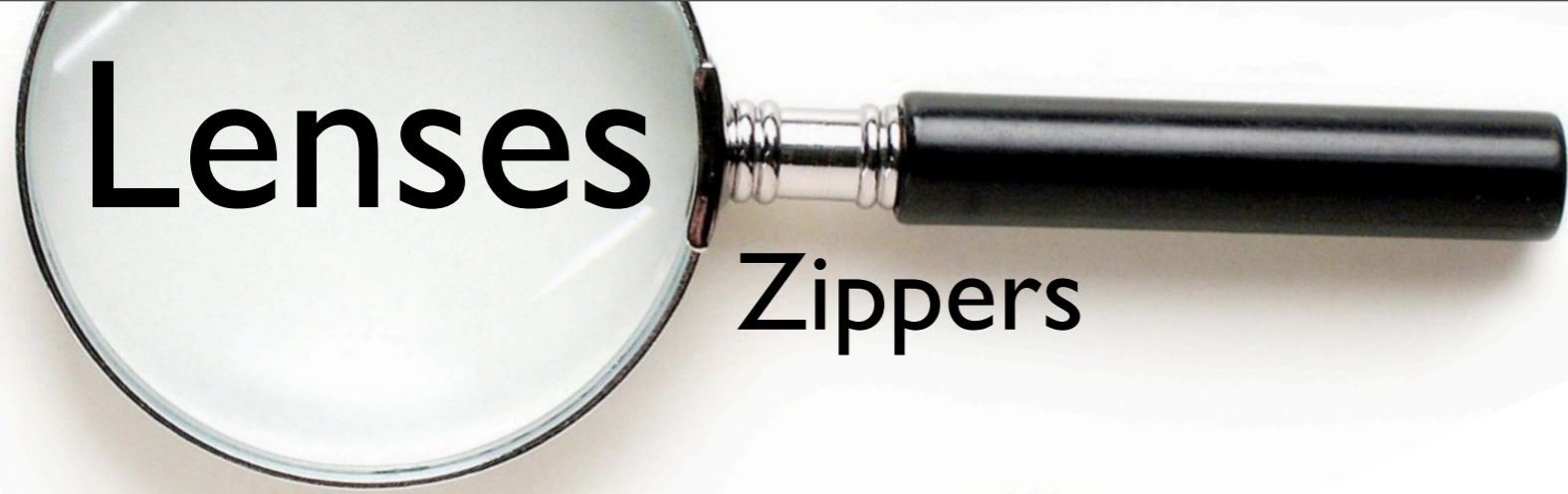
```
instance Plated (Tree a) where
  plate f (Node a as) = Node a <$> traverse f as
```

```
template :: (Data a, Typeable b) => Simple Traversal a b
uniplate :: Data a => Simple Traversal a a
biplate :: (Data a, Typeable b) => Simple Traversal a b
```

```
children :: Plated a => a -> [a]
children = toListOf plate
```

```
rewriteOf :: Simple Setter a a -> (a -> Maybe a) -> a -> a
rewriteOf l f = go where go = transformOf l (\x -> maybe x go (f x))
```

```
contextsOf :: SimpleLensLike (Bazaar a a) a a -> a -> [Context a a a]
holesOf :: LensLike (Bazaar c c) s t c c -> s -> [Context c c t]
paraOf :: Getting [a] a b a b -> (a -> [r] -> r) -> a -> r
partsOf :: LensLike (Bazaar c c) s t c c -> Lens s t [c] [c]
unsafePartsOf :: LensLike (Bazaar c d) s t c d -> Lens s t [c] [d]
```



Zippers

Lenses

Zippers

```
zipper ("hello", "world")
  % down _1
  % fromWithin traverse
  % focus .~ 'J'
  % rightmost
  % focus .~ 'y'
  % rezip
("Jelly", "world")
```

```
zipper :: a -> Top :> a
up :: (a :> b :> c) -> a :> b
down :: Simple Lens b c -> (a :> b) -> a :> b :> c
within :: Simple Traversal b c -> (a :> b) -> Maybe (a :> b :> c)
fromWithin :: Simple Traversal b c -> (a :> b) -> a :> b :> c
```

```
left, right :: (a :> b) -> Maybe (a :> b)
lefts, rights :: Int -> (h :> a) -> Maybe (h :> a)
leftmost, rightmost :: (a :> b) -> a :> b
```

```
save :: (a :> b) -> Tape (a :> b)
restore :: Tape (h :> a) -> Zipped h a -> Maybe (h :> a)
```

Lenses



References

- J. Nathan Foster, Alexandre Pilkiewicz, and Benjamin C. Pierce. [Quotient Lenses](#). *ACM SIGPLAN International Conference on Functional Programming (ICFP)*, Victoria, British Columbia, September, 2008.
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- - [lenses, fclabels, data-accessor - which library for structure access and mutation is better](#). Stack Overflow
- R. O'Connor. [Functor is to Lens as Applicative is to Biplate](#). arXiv: 1103.2841
- T. van Laarhoven. [Talk on Lenses](#) <http://twanvl.nl>

Lenses

Questions

Any
Questions



Extra Slides

Lenses

The Power is in the Dot

```
fmap      :: Functor f      => (a -> b) -> f a -> f b
fmapDefault :: Traversable f => (a -> b) -> f a -> f b
fmapDefault f = runIdentity . traverse (Identity . f)
```

```
foldMap      :: (Foldable f, Monoid m)    => (a -> m) -> f a -> m
foldMapDefault :: (Traversable f, Monoid m) => (a -> m) -> f a -> m
foldMapDefault f = getConst . traverse (Const . f)
```

```
traverse :: (Traversable f, Applicative m) => (a -> m b) -> f a -> m (f b)
```

```
newtype Const m a = Const m
```

```
instance Functor (Const m) where
  fmap _ (Const m) = Const m
```

```
instance Monoid m => Applicative (Const m) where
  pure _ = Const mempty
  Const m <*> Const n = Const (m <> n)
```

Lenses

Isomorphism-Based Lenses

```
data a ↔ b
```

```
= Iso (a -> b) (b -> a)
```

```
newtype Lens a b =
```

```
forall c. Lens (a ↔ (c, b))
```

Lenses

Isomorphism-Based Lenses

```
data a ↔ b
```

```
= Iso (a -> b) (b -> a)
```

```
newtype Lens a b =
```

```
forall c. Lens (a ↔ (c, b))
```

```
(%%~) :: Functor f =>
```

```
    Lens a b -> (b -> f b) -> a -> f a
```

```
Lens (Iso to fro) f a = case to a of
```

```
(c, b) -> fro . ((,) c) <$> f b
```

Lenses

Isomorphism-Based Lenses

```
data a ↔ b
```

```
= Iso (a -> b) (b -> a)
```

```
newtype Lens a b =
```

```
forall c. Lens (a ↔ (c, b))
```

```
(%%~) :: Functor f =>
```

```
    Lens a b -> (b -> f b) -> a -> f a
```

```
Lens (Iso to fro) f a = case to a of
```

```
(c, b) -> fro . ((,) c) <$> f b
```

Lenses

van Laarhoven Lenses

```
type Lens a b = forall f.
```

```
  Functor f => (b -> f b) -> a -> f a
```

```
(%%~) :: Lens a b -> (b -> f b) -> a -> f a
```

```
(%%~) = id
```

Lenses

van Laarhoven Lenses

```
type Lens a b = forall f.
```

```
  Functor f => (b -> f b) -> a -> f a
```

```
(%%~) :: Lens a b -> (b -> f b) -> a -> f a
```

```
(%%~) = id
```

```
(.~) :: Lens a b -> b -> a -> a
```

```
l .~ b = runIdentity . l (Identity . const b)
```

```
(^. ) :: a -> Lens a b -> b
```

```
a ^. l = getConst (l Const a)
```


Lenses

van Laarhoven Lens Families

```
type Lens a b c d = forall f.
```

```
  Functor f => (c -> f d) -> a -> f b
```

```
(%%~) :: Lens a b c d -> (c -> f d) -> a -> f b
```

```
(%%~) = id
```

```
(.~) :: Lens a b c d -> d -> a -> b
```

```
l .~ b = runIdentity . l (Identity . const b)
```

```
(^. ) :: a -> Lens a b c d -> c
```

```
a ^. l = getConst (l Const a)
```

Lenses

van Laarhoven Lens Families

```
type Lens a b c d = forall f.
```

```
  Functor f => LensLike f a b c d
```

```
type LensLike f a b c d =
```

```
  (c -> f d) -> a -> f b
```

```
(%%~) :: Lens a b c d -> (c -> f d) -> a -> f b
```

```
(%%~) = id
```

```
(.~) :: Lens a b c d -> d -> a -> b
```

```
l .~ b = runIdentity . l (Identity . const b)
```

```
(^. ) :: a -> Lens a b c d -> c
```

```
a ^. l = getConst (l Const a)
```

Lenses

van Laarhoven Lens Families

```
type Lens a b c d = forall f.
```

```
  Functor f => LensLike f a b c d
```

```
type LensLike f a b c d =
```

```
  (c -> f d) -> a -> f b
```

```
(%%~) :: Lens a b c d -> (c -> f d) -> a -> f b
```

```
(%%~) = id
```

```
(.~) :: Lens a b c d -> d -> a -> b
```

```
l .~ b = runIdentity . l (Identity . const b)
```

```
(^. ) :: a -> Lens a b c d -> c
```

```
a ^. l = getConst (l Const a)
```

Lenses

van Laarhoven Lens Families

```
type Lens a b c d = forall f.
```

```
  Functor f => LensLike f a b c d
```

```
type LensLike f a b c d =
```

```
  (c -> f d) -> a -> f b
```

```
(%%~) :: Lens a b c d -> (c -> f d) -> a -> f b
```

```
(%%~) = id
```

```
(.~) :: LensLike Identity a b c d -> d -> a -> b
```

```
l .~ b = runIdentity . l (Identity . const b)
```

```
(^. ) :: a -> LensLike (Const c) a b c d -> c
```

```
a ^. l = getConst (l Const a)
```


Lenses

Indexed Lenses, Traversals, etc.

```
class Indexed i k where
  index :: ((i -> a) -> b) -> k a b

type Indexable i a b = forall k. Indexed i k => k a b

instance Indexed i (->) where
  index f = f . const

newtype Index i a b = Index { withIndex :: (i -> a) -> b }

instance i ~ j => Indexed i (Index j) where
  index = Index

indexed :: Indexed Int k => LensLike (Indexing f) a b c d -> k (c -> f d) (a -> f b)
```

Lenses

Indexed Lenses, Traversals, etc.

```
type Overloaded k f a b c d = k (c -> f d) (a -> f b)
```

```
-- turn a normal traversal or setter into an indexed traversal or setter.
```

```
indexed :: Indexed Int k => LensLike (Indexing f) a b c d -> k (c -> f d) (a -> f b)
```

```
class At k m | m -> k where
```

```
  at :: k -> SimpleIndexedLens k (m v) (Maybe v)
```

```
traverseAt :: At k m => k -> SimpleIndexedTraversal k (m v) v
```

```
value :: (k -> Bool) -> SimpleIndexedTraversal k (k, v) v
```

```
iwhereOf :: (Indexed i k, Applicative f) =>
```

```
  Overloaded (Index i) f a b c c -> (i -> Bool) -> Overloaded k f a b c c
```

... and there are similarly a ton of combinators for these, too...



Lenses

Overview



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